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Graded Homework 2

¬ ∴ ∀(x) ∃(x) **≡** ∧

1. J – get a job C – buy a new car H – buy a new house

J 🡪(C ∧ H)

¬H

∴ ¬J

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| 1. | Hypothesis | J 🡪(C ∧ H) |
| 2. | Hypothesis | ¬H |
| 3. | Modus Tollens | ¬(C V H) |
| 4. | De Morgan’s Law | ¬C ∧ ¬H |
| 5. | Commutative Law | ¬H ∧ ¬C |
| 6. | Simplification | ¬H |

1. P – practices hard B – plays badly S - players

∀(x): P(x) V B(x) V (P(x)∧B(x))

∃(x): ¬P(x)

∴∃(x) B(x)

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| 1. | Hypothesis | ∃(x): ¬P(x) |
| 2. | Existential instantiation | (S is an particular) element ∧ P(S) |
| 3. | Hypothesis | ∀(x): P(x) V B(x) V (P(x)∧B(x)) |
| 4. | Universal Instantiation | P(S) |
| 5. | Simplification | B(S) |
| 6. | Conjunction | ∃(S): ¬P(S) |
| 7. | Universal Generalization | ∀(S): B(S) |

1. An erroneous proof of the statement “If n is an even integer, then n2is  
   an even integer.” is:  
   Proof. m = 10 is an even integer since 10 = 2 ·5. m2= (10)2= 100,  
   which is even since 100 = 2 ·50. Therefore, if n is an even integer, then  
   n2 is an even integer.

This proof is incorrect because it **only** shows that m2 is even when m=10.

Proof: let n be an even integer, n = (2k), then n2 = (2k)2 = 4k2, which simplified is 2(2k2) making n2 an even integer.

1. An erroneous proof of the statement “The difference between two odd  
   numbers is even.” is:  
   Proof. Let x and y be two odd integers. Since x is odd, ∃j ∈Z, x = 2j +1.  
   Since y is odd, ∃k ∈Z, y = 2k + 1. Since both x and y are odd, x −y must  
   be even. Therefore the difference between two odd integers is even.

This proof is incorrect because it says that if x and y are both odd integers, then x-y = **even**, which is not true.

Proof: let x and y be two odd integers, x = (2j+1) and y = (2k+1). Then x-y = (2j+1) – (2k+1) = 2j-2k, which simplifies to 2(j-k) making an even integer.

1. Definition 1. The absolute value of a real number x is defined to be  
   |x|= −x if x < 0, and |x|= x if x ≥0.  
   Theorem 1. ∀x, y ∈R, |x −y|= |y −x|

Proof by Cases:

Let x, y be real numbers.

1. If x >= y then x-y >= 0. So,

|x-y| = x-y = y-x = |y-x|.

1. X < y means x-y < 0. So,

|x-y| = -(x-y) = y-x = |y-x|.

In both cases, |x-y| = |y-x|, solving the proof.